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## MEETING ON ELECTORAL SYSTEMS IN TUNISIA

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# APPLIED ASPECTS OF SIMPLE SYSTEMS[1] FOR PROPORTIONAL REPRESENTATION (SPR)[2]

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[1] For the purpose of this presentation, the term 'simple system' means a system where the voter marks his or her choice on the ballot with one tick.

[2] The considerations below are mostly focused on systems for proportional representations (SPR), since this meeting is dedicated to the current SPR in Tunis. It is regulated by Decree 35 of 10 May 2011 'On the Election of the National Constituent Assembly' as revised and completed by Decree 72 of 3 August 2011, Articles 31-36. The Tunisian system is a typical regional SPR, where candidates are included in closed constituency candidate lists, without a representation threshold. Lists compete for the seats in the constituency where they are registered. Seats are allocated to eligible lists by the method Hare-Niemeyer-Hamilton. The total number of seats contested in Tunisia on 23 October 2011 was 217. A summary of state wide results, mostly focused on the parties which won seats in the Tunisian parliament, is available at http://www.tunisia-live.net/2011/11/14/tunisian-election-final-results-tables/.

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## I.    INTRODUCTION

1.  The electoral system[3] is the mathematical tool used to translate votes cast by popular vote for competing electoral subjects[4] into seats in the body elected. The *majoritarian* systems for representation are usually perceived as a competition between individuals. The systems for *proportional* representation (SPR) are characterized by competition between party platforms represented by lists of candidates.[5] In addition, there are also *mixed* electoral systems that *combine* elements of majoritarian systems and SPRs with a view to establish a compromise between the advantages the above major groups.

2.  The choice of an electoral system is considered a sovereign act, which should ideally be a result from a broad agreement on a number of political issues between key stakeholders. Issues include how proportional (or inclusive) the SPR should be, would the SPR provide for positive discrimination of some social segments such as minorities and the less represented gender, and whether voters will be granted the right to reorder the lists of candidates through their votes.

3.  Prior to clarifying these issues, it might be premature to discuss specific aspects of a particular SPR such as constituencies, thresholds and allocation methods. It is key to understand that these factors influence the performance of the system in a mutually dependent manner, rather than independently.

4.  Any electoral system has to provide unambiguous answers with regard to which electoral subjects are eligible to get seats, how many seats will get each eligible subject and who are the individuals that will fill the seats? The concrete answers to these questions may result in quite different SPRs.

## II.    INTERNATIONAL STANDARDS

5.  In relation to electoral systems, international standards for democratic elections are mostly limited to Article 25 of the UN International Covenant for Civil and Political Rights and its interpretation by Paragraphs 17 and 21 of General Comment 25 of the UN Human Rights Committee.[6] On a similar note, Article 3 of Protocol 1 to the European Convention of Human

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[3]    An excellent first course in electoral systems is provided by Robert Newland's book *"Comparative Electoral Systems"*, The Arthur McDougall Fund, London, 1982.

[4]    In this context, electoral subjects may be lists of candidates of parties and/or pre-electoral coalitions, and individual candidates running as 'independents', as required by international standards for democratic elections, Section 2 'International Standards' (below).

[5]    Lists can be (a) *state wide and regional*, (b) *open and closed*: lists are open if voters are allowed to add and/or delete names in candidates lists when they vote, otherwise lists are closed; and (c) *flexible and rigid*: lists are flexible if voters are allowed to change the order of the candidates' names in the lists when they vote, otherwise lists are rigid. Usually, the number of names on a list of candidates does not exceed the number of seats returned by the electoral district where the list has been registered for the election.

[6]    The United Nations' International Covenant on Civil and Political Rights, http://www2.ohchr.org/english/law/ccpr.htm. The General Comments are available at http://www.unhchr.ch/tbs/doc.nsf. Paragraph 17 of General Comment 25 provides: 'The right of persons to stand for election should not be limited unreasonably by requiring candidates to be members of parties or of specific parties. […]' Paragraph 21 of General Comment 25 provides: 'Although the Covenant does not impose any particular electoral system, any system operating in a State party must be compatible with the rights protected by article 25 and must guarantee and give effect to the free expression of the will of the electors. The principle of one person, one vote, must apply, and within the framework of each State's electoral system, the vote of one elector should be equal to the vote of another. The drawing of electoral boundaries and the method of allocating votes should not distort the distribution of voters or discriminate against any group and should not exclude or restrict unreasonably the right of citizens to choose their representatives freely.'

Rights upholds the right of free elections, 'under conditions which will ensure the free expression of the opinion of the people in the choice of the legislature'.[7]

6.   The Council of Europe's European Commission for Democracy through Law (the Venice Commission, VC) has developed a Code[8] which provides guidance on the design of specific elements of electoral systems in line with international standards for democratic elections.


## III.    PROPORTIONALITY

7.   SPRs have been developed in order to ensure inclusiveness of (political) minority opinions, in proportion to their influence in society, close to and/or on election day. As seats allocated are an integer number, proportionality is in most cases approximate. *Ideal proportionality* requires that the percentages of the votes cast for all electoral subjects are the same as the percentages of seats allocated to the respective subjects within a given constituency or set of constituencies.

8.   If *n* electoral subjects contested an election held under a SPR, a possible formula for calculating an *a posteriori* estimate, *Δ*, for the *departure from ideal proportionality*, or *disproportionality* is:

$$\Delta = [(v_1 - s_1)^2 + (v_2 - s_2)^2 + \ + (v_n - s_n)^2]^{1/2} \ [\%] \geq 0,$$

where $v_1$ and $s_1$ are the percentages of the votes received by and the seats allocated to subject "*1*", $v_2$ and $s_2$ are the respective percentages for subject "*2*", …, $v_n$ and $s_n$ are the percentages for subject "*n*".[9] For ideal proportionality, *Δ* = 0. Large values of *Δ* indicate a large departure from ideal proportionality.

9.   Generally, some of the competing subjects fail to get even a single seat. Notwithstanding, their participation should be reflected in the calculation of *Δ* and on occasion such contributions could be substantial. If one is interested only in the proportionality of the results of those subjects which got at least one seat, one could ignore the contribution of the subjects which failed in getting a seat and calculate the *truncated disproportionality*, $\Delta^*$, where

$$\Delta \geq \Delta^* \geq 0.$$

*Example 1*

Let us have four competing subjects, which received 40%, 30%, 25% and 5% of the valid votes, and 45%, 33%, 22% and 0% of the seats, respectively. Then, one has:

$v_1 = 40$        $v_2 = 30$        $v_3 = 25$        $v_4 = 5$, $v_1 + v_2 + v_3 + v_4 = 100$        and
$s_1 = 45$        $s_2 = 33$        $s_3 = 22$        $s_4 = 0$, $s_1 + s_2 + s_3 + s_4 = 100$.

Then:

$\Delta = [(40 - 45)^2 + (30 - 33)^2 + (25 - 22)^2 + (5 - 0)^2]^{1/2} = [25 + 9 + 9 + 25]^{1/2} = [68]^{1/2} \approx 8.25\%$,    and
$\Delta^* = [25 + 9 + 9]^{1/2} = [43]^{1/2} \approx 6.56\%$.

10.   Often, a political compromise between the inclusiveness of political opinions and the stability of government can be achieved by sacrificing a measure of proportionality. Under the same electoral system, the higher the number of seats to allocate, the less the disproportionality would be.

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[7]       Available at http://www.echr.coe.int.

[8]       Council of Europe's Venice Commission 'Code of Good Practice in Electoral Matters', Part I 'Principles of Europe's Electoral Heritage', Section I.2 'Equal Suffrage', http://www.venice.coe.int/docs/2002/CDL-AD(2002)023rev-e.pdf.

[9]       This formula utilizes what is known in mathematical analysis as the Euclidian norm or (abstract) Euclidian distance.

## IV.    CONSTITUENCIES[10]

### A.    Background

11.  SPRs are based on one or more constituencies (electoral districts) spanning the territory of the state. Constituencies return a pre-determined number of elected representatives each and are referred to as multi-seat constituencies. While borders of constituencies often coincide with the administrative regional borders, there may be constituencies which diverge from regional administrative borders.[11]

12.  When a SPR is implemented in a single state wide constituency which returns all members of the elected body, it is known as a state wide SPR and yields results that are usually closest to ideal proportionality. Alternatively, when a SPR is implemented in a set of several constituencies, it is known as a regional SPR.

### B.    Natural Threshold

13.  In a multi-seat constituency, in order for a subject to get a seat without other restrictions, it has to receive a minimum number of votes. This number is called a *natural threshold*. An upper bound[12] for the value of the natural threshold, $T_1^*$, can be estimated on the basis of the number of seats, $M$, returned by the multi-seat constituency and the total number of valid votes, $V$, cast in the constituency:

$$T_1^* = V / (M+1) + 1, \text{ or } T_1^*/V = 1/(M+1) + 1/V = 1/(M+1) + \delta, \delta = 1/V.$$

14.  This is the Hagenbach-Bischoff formula.[13] In percentages, $T_1$ can be estimated as:

$$T_1 = T_1^*/V * 100 \, [\%] = [1/(M+1)] * 100 + \varepsilon \, [\%], \varepsilon = \delta * 100.$$

15.  The lesser number of seats that a constituency returns, the higher $T_1$ is and it is less likely for subjects with a low number of votes to get a seat.

*Example 2*

16.   If a multi-seat constituency returns 100 seats, then $T_1 = 1 / 101 + \varepsilon = 0.99\% + \varepsilon$. Consequently, if a party has received at least a bit less than 1% of the vote, it will get at least one seat. Importantly, a party could get at least one seat with less than 1% of the vote under specific circumstances depending on the concrete numbers of votes and/or legal provisions. If the constituency returns 10 seats, $T_1 = 9.09\% + \varepsilon$ and a party will get at least one seat if it has more than one eleventh of the vote in the constituency. A multi-seat constituency, which returns the lowest number of seats, is a constituency returning two seats. In this case, $T_1 = 33.33\% + \varepsilon$ and a party will get at least one seat if it has more than one third of the vote in the constituency. Obviously, if two subjects have got at least $T_1$ percent of the vote each, they will get the two seats, while any other subject cannot have received this number of votes or more. A similar conclusion is valid for any number of seats returned by the constituency. In a one-member constituency, $T_1 = 50\% + \varepsilon$, which is the absolute majority of the vote in the constituency.[14]

17.   A typical regional SPR for the election of a state institution is based on a number of constituencies, which return different numbers of members of the elected body and have

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[10]      Constituencies are often referred to as 'electoral districts'.
[11]      See also the VC Code, Section I.2.2ii.
[12]      The upper bound is an estimated value higher than the actual value of the quantity in question.
[13]      This formula resembles closely the Droop quota, known as the least quota that allows to allocate not more than the M seats available, please see also Section 6.2.1 'Quota (Remainder) Methods', below. Its rational is that in order for a subject to get at least one seat in the constituency, it should accrue at least one vote more than the Droop quota,
[14]      Please see also http://www.venice.coe.int/docs/2008/CDL-AD(2008)037-e.pdf.

therefore different natural thresholds. This could always inspire perceptions that parties in the individual constituencies compete under different conditions, in particular if the natural thresholds of the constituencies vary broadly. In addition, as seats are allocated within each constituency independent from any other constituency, while disproportionality in each constituency could approach the possible minimum, disproportionality at state level could be higher.

### C.     Equality of the Vote

18.    In at least one chamber of a state parliament, each member should represent an approximately equal number of citizens, which is called a *norm of uniform representation, U*. According to the VC Code, the permissible departure of this norm should not exceed 10%.[15] This will guarantee the equality of the vote – that the vote of each voter has approximately equal power as the vote of any other voter – as ultimately required by international standards.

*Example 3*

19.  In a state wide constituency which returns all 100 members of the lower chamber of the legislature of a state with population of 4.5 million, the norm of uniform representation is:
$$U = 4,500,000 \,/\, 100 = 45,000.$$

20.  When the state is divided into $S$ constituencies, the number of members returned by the constituencies, or equivalently – the number of seats allocated to the constituencies, should be determined in such a way so that it is guaranteed, that in each constituency the norm of representation, $U_j$, $j$ = 1,2, …, $S$, is as close as possible to the state wide norm $U$, or that the *equality of the vote* is respected. This means that the numbers of members returned by the constituencies is proportional to the population numbers of the constituencies. The allocation of seats to constituencies in proportion to the respective population numbers can be accomplished by any allocation method.[16]

21.    It is possible that low populated constituencies are allocated disproportionally higher numbers of seats for various political considerations, e.g. establishing a measure of territorial representation or ensuring that some of the constituencies which are least populated be guaranteed representation.[17] Typically, this happens when legislation provides that all constituencies are allocated initially equal numbers of seats in the state legislature and after this – the remaining seats are allocated proportionally to the respective population numbers. In this case, the regional SPR might be at odds with international standards because the equality of the vote is likely to not be fully respected. This is due to the fact that the votes of the voters in less populated constituencies 'carry more weight' than the votes of the voters of more populated constituencies or, alternatively, members returned by such constituencies represent lower numbers of population.

## V.     REPRESENTATION THRESHOLD

### A.     Background

22.  Often, legal provisions prescribe that the subjects eligible to get seats must have received at least $T_2$ per cent of the valid votes, either within each constituency or state wide. The quantity $T_2$ is called *representation* (or *legal*) *threshold*, which is usually defined in percentages by law.

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[15]        The VC Code, Section I.2.2.iv.
[16]        Please see Section 6 'Seat Allocation Methods' (below).
[17]        See also Example 7(a) (below).

The magnitude of $T_2$ is an indicator of the will of the legislator with regard to the intended balance between inclusiveness and stability.[18]

*Example 4*

23. Parties A, B and C have contested an election on the basis of state wide lists, with five per cent representation threshold, $T_2 = 5\%$. The parties have won $V_A = 200,000$, $V_B = 20,000$ and $V_C = 300,000$ votes, respectively. The total number of valid votes cast, $V$, is:
$$V = V_A + V_B + V_C = 520,000$$

and the number of votes $V_{T2}$ corresponding to the representation threshold $T_2$ is:
$$V_{T2} = V * T_2 = 0.05*520,000 = 26,000.$$

24. Therefore, $V_B < V_{T2}$ and party B is excluded from the seat allocation.

## B.    Impact on Proportionality

25. While $T_2$ always aims to determine which subjects will be eligible for seat allocation, it can have different impact on the proportionality of the outcome depending on the manner that it is defined and applied.

26. In a SPR implemented in a single state wide constituency, $T_2$ will simply determine which subjects are eligible for seat allocation and eligible subjects will get numbers of seats proportional to their state wide vote totals. In a SPR implemented in several multi-seat constituencies the impact of $T_2$ may vary broadly as it may or may not interact with $T_1$.

27. If the representation threshold $T_2$ is determined at constituency level as a percentage of the constituency wide vote total (which would generally differ from one constituency to another), in order for it to have any effect on the seat allocation in a given constituency, it should be higher that the respective natural threshold $T_1$.

*Example 5*

28. Let a constituency return 19 members. Then, according to Example 2, $T_1 = 5\% + \varepsilon$ and a subject will get at least one seat if it has more than 5% of the vote in the constituency. If the representation threshold is $T_2 = 3\%$, it will (most likely) have no impact on the seat allocation, because if a subject has received 3.5% of the valid votes in this constituency it will still (most likely) not have enough votes to qualify for a seat there due to the value of $T_1$. However, if the constituency returns 34 seats, then $T_1 = 2.86\% + \varepsilon$ and the subject with 3.5% of the vote will be eligible for seat allocation and will get at least one seat.

29. However, if $T_2$ is determined at state level as a percentage of the state wide total of the votes, the effect of $T_2$ will depend on the level at which the seat allocation between the eligible parties takes place. There are two possibilities:

  (a) Seats may be allocated to eligible subjects proportionally to their votes within each constituency taking into account the impact of $T_1$, independent from any other constituency.
  (b) Seats may be allocated to eligible subjects proportionally to their vote totals state wide. After that, the seats allocated to each eligible subject state wide are allocated to the constituencies in proportion to the respective subject's votes in the constituencies.[19]

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[18]    Legal provisions in a few states stipulate that if the 'largest' subject after an election cannot establish absolute majority in parliament, it receives a number of 'premium' seats with a view to reach absolute majority and enhance stability.

### C.    Turnout Threshold

30.    Some states have chosen to introduce into legislation, a requirement that a given percentage of the registered voters, $T_3$, participate[20] in the election for the election to be valid. Such states have argued that a minimum participation in the election underscores its "legitimacy". If the percentage of participation is less than the *turnout threshold $T_3$*, the election is considered invalid and has to be repeated from the beginning. Such legislation opens the possibility to an endless cycle of failed elections and invites electoral malfeasance.

## VI.    SEAT ALLOCATION METHODS

### A.    General Formulation

31.    Generally, the mathematical problem is to distribute (or allocate) a total number of *X* identical spheres (or any other identical objects) to *Y* baskets of different volumes, subject to the following conditions:

   (a) The numbers of spheres allocated to each basket are proportional to the volumes of the baskets;
   (b) No sphere remains unallocated; and
   (c) Allocation of spheres to baskets is conducted through a final number of steps.

32.    In this context, the SPRs referred to in the previous sections have the following interpretations, respectively:

   (a) *Section 4.3 'Equality of the Vote'.* The baskets are the different constituencies, the volume of each basket is the number of population of a given constituency, and the spheres are the members (or seats) that each constituency will have to return. The numbers of seats are proportional to the size of the population in each constituency.
   (b) *Section 5.2 'Impact on Proportionality', paragraphs 2, 3 and 5(a).* The baskets are the individual subjects. The volume of each basket is the number of votes cast for the respective subject in a given constituency, while the spheres are the seats allocated to the subject proportionally to the votes cast.
   (c) *Section 5.2 'Impact on Proportionality', paragraph 5(b), second sentence.* The baskets are the individual constituencies, the volume of the basket is the number of votes cast for the subject in the specific constituency, while the spheres are the seats allocated to each subject, that have to be allocated to the individual constituencies proportionally to this subject's votes cast in the respective constituency.

33.    If the seat allocation to subjects takes place for each individual constituency, the subject with the highest number of votes in a given constituency has some advantage which may be of different magnitude. For subjects with considerable advantage statewide (throughout the constituencies), this advantage would be "multiplied" by the number of the constituencies. This effect can further be augmented if the constituencies return low numbers of members (or have high "natural" thresholds).

### B.    Allocation Methods

34.    There are two major groups of methods to allocate seats within a constituency proportionally to the votes cast for the parties contesting the election in this constituency. These are the *quota* (or *remainder*) methods, and the *quotient* (or *divisor*) methods, with the quota

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[19]        This SPR is known as 'double proportional' and in terms of numbers of seats allocated to eligible subjects it is equivalent to a state wide SPR.

[20]        Participation means that the voter has shown up in the polling station and has received a ballot.

methods reducing somewhat the advantages of the subjects with high numbers of votes.[21] Proper implementation of allocation methods requires an unambiguous legal regulation how equal remainders or quotients are to be treated. It is also possible to allocate some of the seats in a given constituency through a quota method to enhance inclusiveness and the remaining seats – through a quotient method.

### 1. Quota (Remainder) Methods

35.   In the context of Section 6.1 'General Formulation', the quota is the average volume needed for one sphere and in concrete applications it is measured in population numbers or votes (depending on the context) per seat. One calculates how many times the quota is contained in the volume of a specific basket; the result of the division is generally a number with an integer and a fractional part. The fractional part is called *remainder*. The initial number of spheres allocated to a specific basket is equal to the integer part of the ratio "volume of basket divided by quota". If there are still spheres to allocate, these are allocated further by giving the first one to the basket with the largest remainder, the second one – to the basket with the second largest remainder, etc.

36.   If $N$ is the number of seats, $V$ is the total number of valid votes and $P$ is the total number of population, the two commonly used quotas are the Hare quota, $H$ and the Droop quota, $D$, defined as

$$H = V/N \text{ or } H = P/N, \qquad D = V/(N+1) \text{ or } D = P/(N+1) \quad \text{and} \qquad D < H.$$

37.   The above ratios are not rounded. The remainder methods, in particular the Hare quota, tend to favor 'small' subjects or constituencies. $D$ is the smallest quota allowing the allocation of not more than the number of seats, N, available. The use of $D$ diminishes the influence of remainders. A comparative example is provided in Example 6(a).

### 2. Quotient (Divisor) Methods

38.   While the starting steps of the individual divisor methods may differ, the methods work in a similar manner. To apply a divisor method, one calculates subsequently the quotients of the volumes of the baskets with a sequence of positive numbers $d_1$, $d_2$, $d_3$, …, which are called *divisors*.[22] After calculating the quotients with the first divisor, the first sphere is put in the basket with the largest quotient. Once this largest quotient has been used, it is replaced with the quotient of the same volume and the next divisor, as shown by the yellow highlighted figures of Example 7. Now, the second sphere is again put in the basket with the largest quotient. The process continues until all spheres have been allocated to the individual baskets accordingly, examples are provided in below. This shows that one has to define the policy through which one allocates the subsequent apple to the respective basket. The most frequently used quotient methods are the methods of D'Hondt with divisors 1, 2, 3, … and Sainte-Laguë with divisors 1, 3, 5 ,… The latter is considered slightly more favorable for 'small' subjects, as illustrated by Example 7 and it could provide for less disproportionality, in particular when the number of seats allocate is higher.

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[21]     According to R. Newland, please see footnote 1, quota methods are not continuous in the sense that if the number of seats allocated is increased by one (in accordance with the same amounts of subjects' votes), it is possible that one of the subjects gets one seat less. However, quotient methods are continuous in the same sense. In a recent publication 'Mathematical Aspects of Electoral Systems' published at a seminar of the American Institute of Physics in 2007, M. M. Konstantinov provides further analysis including paradoxes related to SPRs.

[22]     While generally divisors are real numbers with an integer and fractional components, in the most commonly used quotient methods the divisors are integer numbers.

*Example 6*

39. Five subjects, A, B, C, D and E have contested N seats in an election under a SPR, where the state has been divided into six constituencies, $C_1$, $C_2$, $C_3$, $C_4$, $C_5$ and $C_6$, with respective populations $P_1$, $P_2$, $P_3$, $P_4$, $P_5$ and $P_6$, as follows:

$V_A$ = 200,000          $C_1$: $P_1$ = 300,000          V = 450,000
$V_B$ = 120,000          $C_2$: $P_2$ = 200,000          P = 1,000,000
$V_C$ = 70,000           $C_3$: $P_3$ = 180,000
$V_D$ = 40,000           $C_4$: $P_4$ = 120,000
$V_E$ = 20,000           $C_5$: $P_5$ = 130,000
                         $C_6$: $P_6$ = 70,000

(a) Let N = 100; allocate the contested 100 seats between the five subjects, which have registered state-wide closed lists, by the quota methods of Hare and Droop. Explore the influence of representation thresholds of 3, 5 and 10 %. The two methods produce identical results for higher thresholds and the role of the remainders (red script) with the Droop quota is reduced (less remainders allocate seats).

| Threshold | $T_2$ = 3 % or 13,500 votes | $T_2$= 5 % or 22,500 votes | $T_2$ = 10% or 45,000 votes |
|---|---|---|---|
| Hare quota | H=450,000/100=4,500 | H=430,000/100=4,300 | H=390,000/100=3,900 |
| Droop quota | Q=450,000/101=4,455.45 | Q=430,000/101=4,257.43 | Q=390,000/101=3,861.39 |

| Subject | Votes | Hare | Droop | Hare | | Droop | Hare | | Droop |
|---|---|---|---|---|---|---|---|---|---|
| A | 200,000 | 44.44 **44** | 44.89 **45** | 46.51 | **47** | 46.98 | 51.28 | **51** | 51.79 |
| B | 120,000 | 26.67 **27** | 26.93 **27** | 27.91 | **28** | 28.19 | 30.77 | **31** | 31.08 |
| C | 70,000 | 15.56 **16** | 15.71 **15** | 16.28 | **16** | 16.44 | 17.95 | **18** | 18.13 |
| D | 40,000 | 8.89 **9** | 8.98 **9** | 9.30 | **9** | 9.40 | *** | | *** |
| E | 20,000 | 4.44 **4** | 4.49 **4** | *** | | *** | *** | | *** |
| Seats Total | | 97+3 | 97+3 | | 98+2 | 99+1 | | 98+2 | 100 |

(b) Allocation of seats, S1, S2, S3, S4, S5 and S6, between the six constituencies with population as already described. Use the Hare quota and vary the number of seats, N, to be allocated.

| N | H quota | $P_1$= 300,000 | $P_2$= 200,000 | $P_3$= 180,000 | $P_4$= 120,000 | $P_5$= 130,000 | $P_6$= 70,000 | N |
|---|---|---|---|---|---|---|---|---|
| | | S | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | |
| 10 | 100,000 | 3 | 2 | 1.8 | 1.2 | 1.3 | 0.7 | 8+2 =10 |
| 50 | 20,000 | 15 | 10 | 9 | 6 | 6.5 | 3.5 | 49+1=50 |
| 51 | 19,607.84 | 15.3 | 10.2 | 9.18 | 6.12 | 6.63 | 3.57 | 49+2=51 |
| 100 | 10,000 | 30 | 20 | 18 | 12 | 13 | 7 | 100 |

*Example 7*

40.  The divisors are in the leftmost column. The figures in blue script denote the order in which successive seats are allocated. Figures in red script denote the total number of seats allocated to the respective constituency or party so far. The quotients are in black font. Green highlights illustrate the strategy that the subjects with less seats receives a seat first, when there is a tie. Yellow highlights show sets of quotients used to determine which subject is to get the next seat. In case (a), this would be the 16-th seat allocated as 6-th seat to subject 1, while in (b) – the 2-nd seat allocated as 1-st seat to subject 2. In (a), the smallest subject 6 gets its only seat earlier by the Sainte-Laguë method, as compared to the D'Hondt method.

(a) *Largest quotient* ("allocates an apple only to the basket with the largest quotient") by the Sainte-Laguë method.

| | Subject 1 | Subject 2 | Subject 3 | Subject 4 | Subject 5 | Subject 6 |
|---|---|---|---|---|---|---|
| 1 | 300,000 | 200,000 | 180,000 | 120,000 | 130,000 | 70,000 |
| | (1) (1) | (2) (1) | (3) (1) | (5) (1) | (4) (1) | (7) (1) |
| 3 | 100,000 | 66,666.67 | 60,000 | 40,000 | 43,333.33 | 23,333.33 |
| | (6) (2) | (8) (2) | (9) (2) | (13) (2) | (11) (2) | |
| 5 | 60,000 | 40,000 | 36,000 | 24,000 | 26,000 | 14,000 |
| | (10) (3) | (14) (3) | (15) (3) | | | |
| 7 | 42,857.14 | 28,571.43 | 27,714.86 | 17,142.86 | 18,571.43 | 10,000 |
| | (12) (5) | | | | | |
| 9 | 33,333.33 | 22,222.22 | 20,000 | 13,333.33 | 14,444.44 | 7,777.78 |

…………………………………………………………………………………………………………..

(b) *Largest quotient* ("allocates an apple only to the basket with the largest quotient") or the D'Hondt method. It is considered that this method provides limited advantage to the strongest subject in the constituency. In the context of seat allocation between subjects, the quotient represent the average price of the seats allocated to the subject in question at this stage of the process, e.g. the figure **43,333,33 in the third row of the fifth column** is the average price of the seat that subject 5 would have to pay if it gets 3 seats. Hence, the alternative name of the method – method of highest averages.

| | Subject 1 | Subject 2 | Subject 3 | Subject 4 | Subject 5 | Subject 6 |
|---|---|---|---|---|---|---|
| 1 | 300,000 | 200,000 | 180,000 | 120,000 | 130,000 | 70,000 |
| | (1) (1) | (2) (1) | (3) (1) | (6) (1) | (5) (1) | (11) (1) |
| 2 | 150,000 | 100,000 | 90,000 | 60,000 | 65,000 | 35,000 |
| | (4) (2) | (7) (2) | (9) (2) | (14) (2) | (13) (2) | |
| 3 | 100,000 | 66,666.67 | 60,000 | 40,000 | 43,333.33 | 23,333.33 |
| | (8) (3) | (12) (3) | | | | |
| 4 | 75,000 | 50,000 | 45,000 | 30,000 | 32,500 | 17,500 |
| | (10) (4) | | | | | |
| 5 | 60,000 | | | | | |

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